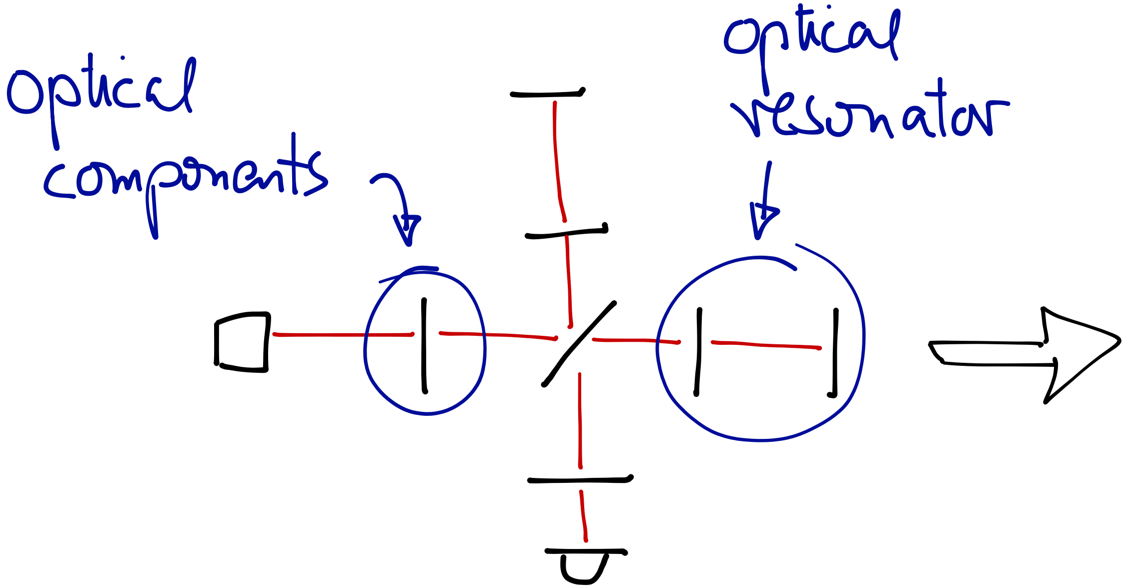
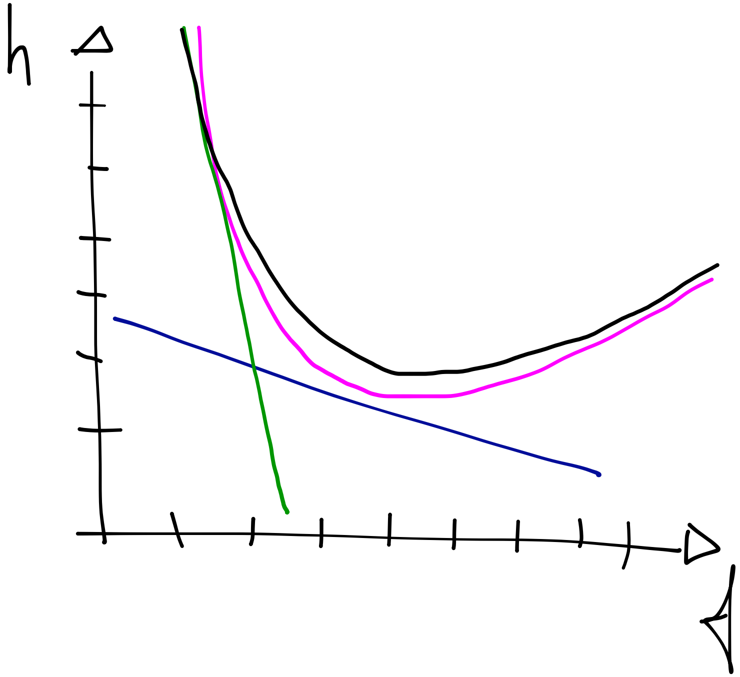
2 OPTICAL COMPS

\+ CAVITY

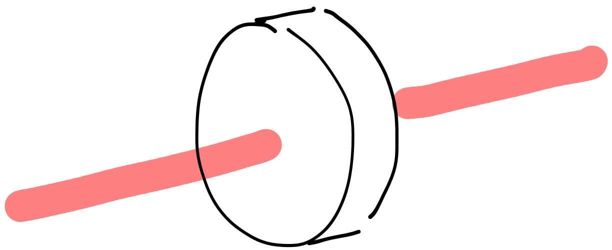


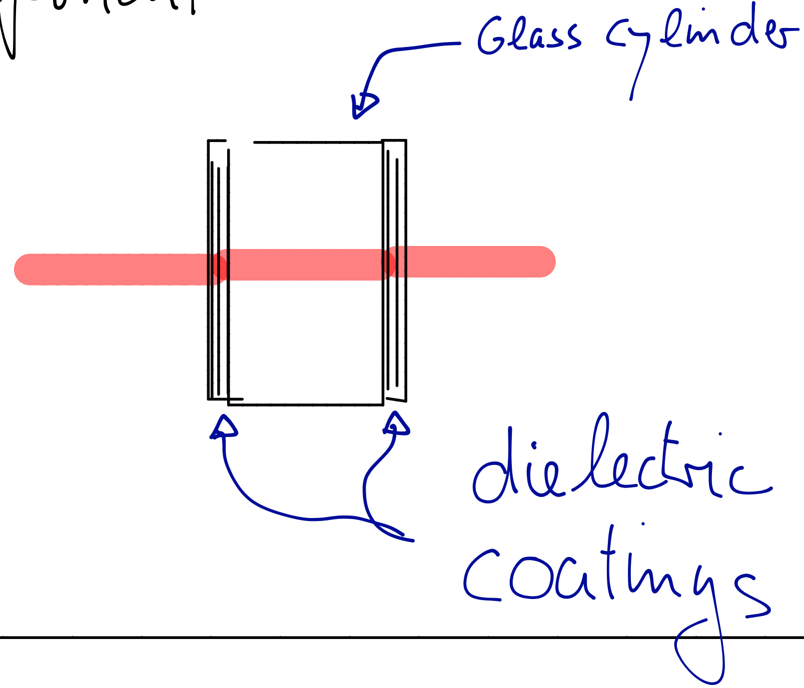


This session:

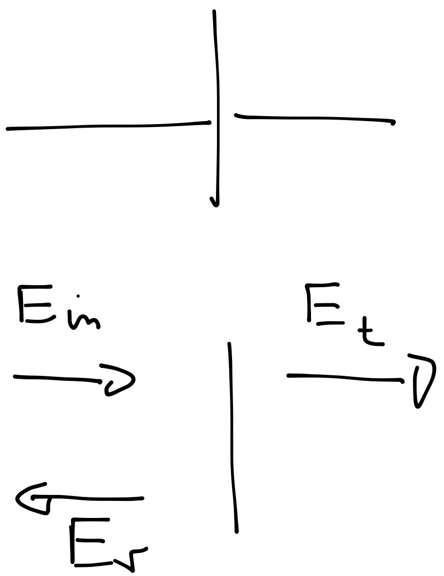
- what happens when the beam hits an optical element
- coupling equations for E fields
- combine multiple optical elements
- example: the optical resonator, also called 'cavity'

Sketch of a mirror component





Simpler sketches and math:

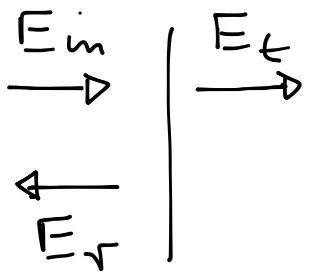


$E_{in} = E_0 e^{i\omega t}$ (mirror at $z=0$)

amplitude and phase can change

this needs to be the same for all 3 fields (continuity)

Coupling equations



$$E_r = r \cdot E_{in}$$

$$E_t = i \cdot t \cdot E_{in}$$

r: amplitude reflectivity

t: amplitude transmission

R: power reflectivity

T: power transmission

$R = r^2$, $T = t^2$, $\boxed{R + T = 1}$

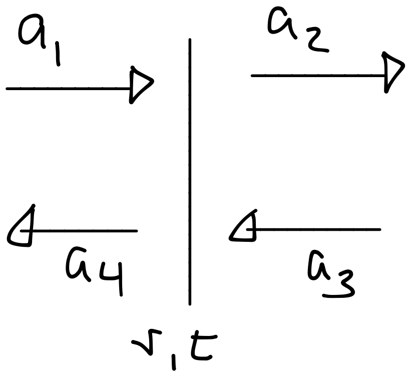
See reading material for the reasons why the following is not allowed:

$$\cancel{E_r = r \cdot E_{in}, \quad E_t = t \cdot E_{in}}$$

We have replaced a complicated unsymmetric object with a simplified symmetric model. We know from experience that the complex details do not matter for most tasks in modelling laser interferometers for GW detectors.

Full coupling equations

mirror



$$a_2 = it\,a_1 + r\,a_3$$
$$a_4 = r\,a_1 + it\,a_3$$

or

$$\begin{pmatrix} a_2 \\ a_4 \end{pmatrix} = \begin{pmatrix} it & r \\ r & it \end{pmatrix} \begin{pmatrix} a_1 \\ a_3 \end{pmatrix}$$

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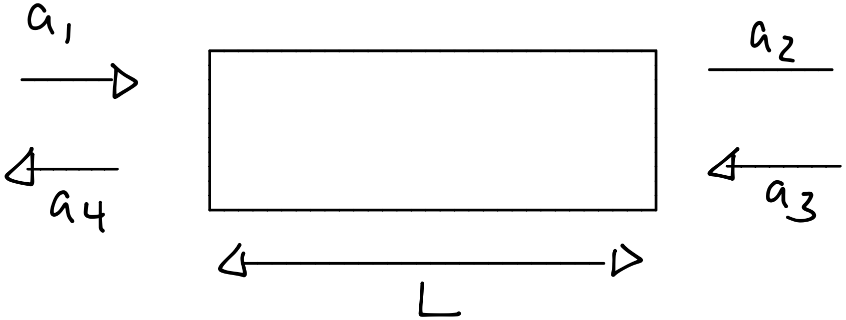
space



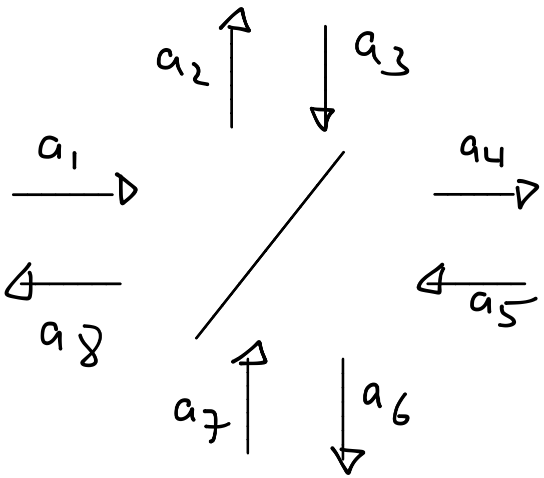
$$a_2 = a_1 \cdot e^{-ikL}$$
$$a_4 = a_3 \cdot e^{-ikL}$$

Note the sign!

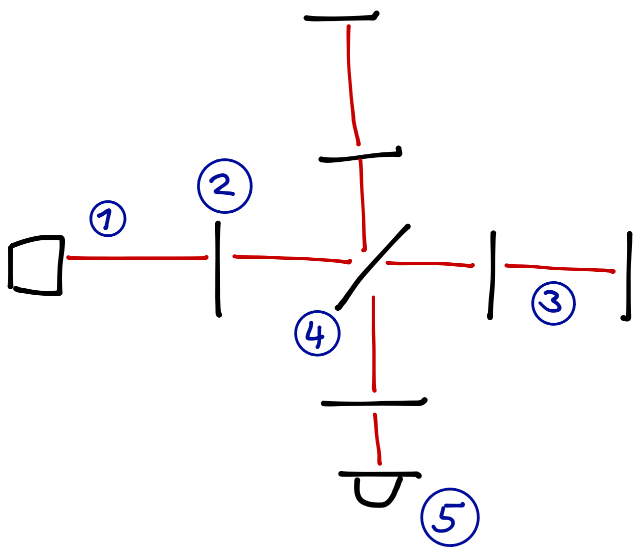
How to remember:
We travel along with the photons!

L2

beam splitter



$$a_2 = r a_1 + it a_7$$
$$a_4 = it a_1 + r a_7$$
$$a_6 = r a_5 + it a_3$$
$$a_8 = it a_5 + r a_3$$



1 beam ✓
2 mirror ✓
3 space ✓
4 beam splitter ✓
5 photo detector ✓

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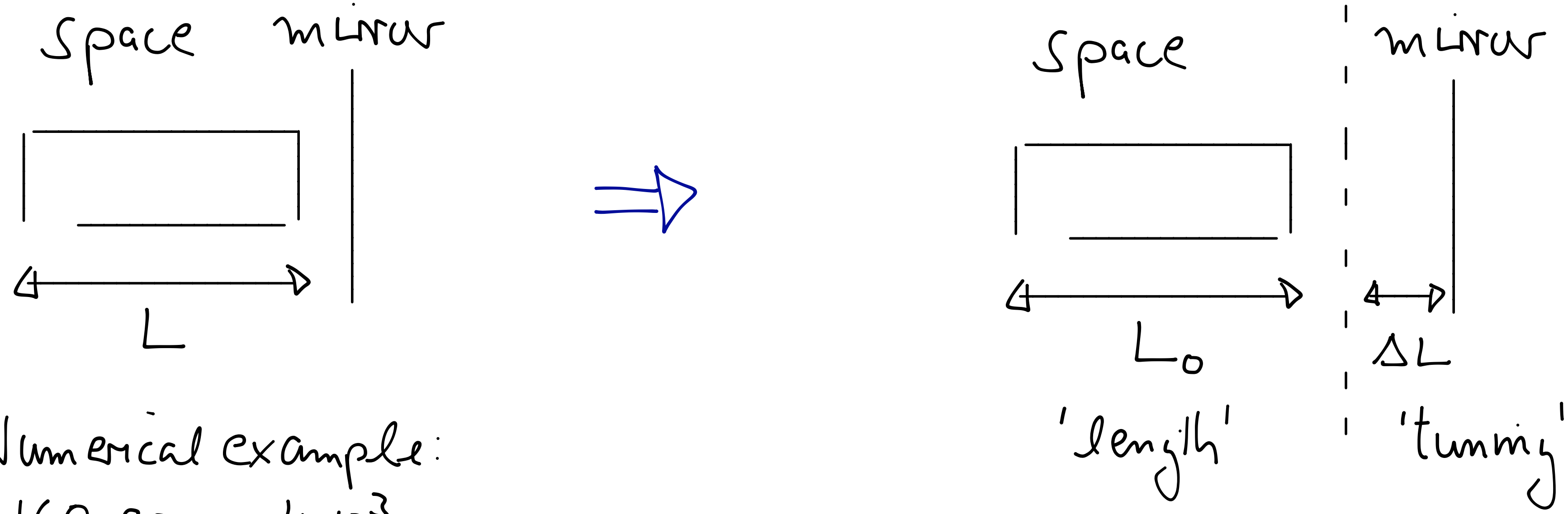
A note on length and positions in the simulations

We will introduce new length parameters



Numerical example:

LIGO arm $= 4 \cdot 10^3$ m

GW induced length change $= 10^{-19}$ m

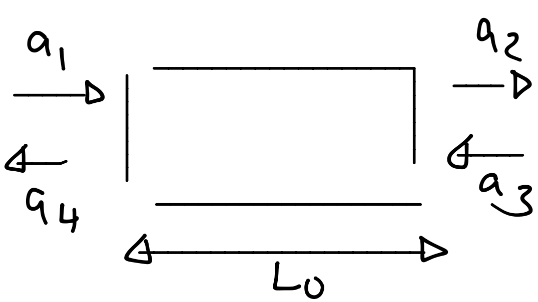
relative change $= 10^{-23}$

accuracy of 'double float' numbers $\approx 10^{-15}$

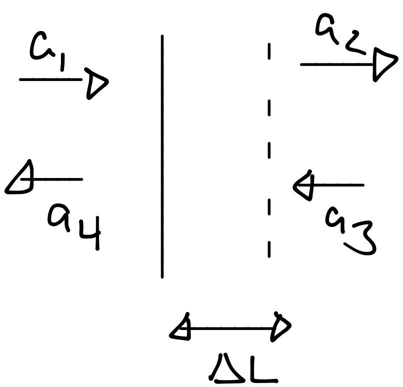
All interferometer signals periodic with $\frac{\lambda}{2}$, i.e. $L = 1000\lambda$ and $L = 100000\lambda$ gives the same result.

We define: $L_0 = N \cdot \lambda$, $\Delta L < \lambda$, $L = L_0 + \Delta L$

For convenience in the numerical model we attach L_0 to spaces and ΔL to mirrors



$$a_2 = a_1 e^{-ikL_0}$$
$$a_4 = a_3 e^{-ikL_0}$$



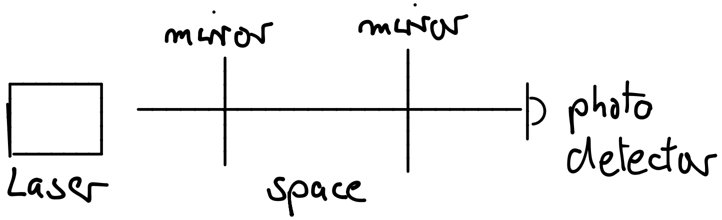
$$a_2 = it a_1 + r e^{-i2k\Delta L} a_3$$
$$a_4 = r \underset{R}{e}^{i2k\Delta L} a_1 + it a_3$$

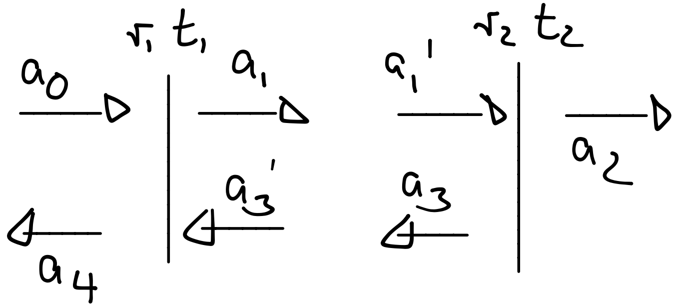
again, note the signs

Not needed for simple analytics but important for using FINESSE later.

(In FINESSE, tuning ΔL is given as $\phi = 360 \cdot \frac{\Delta L}{\lambda}$

Let's try our first interferometer, the optical resonator, also called 'cavity'.





We simply use our coupling equations:

$$\left.\begin{aligned} a_1 &= i t_1 a_0 + r_1 a_3' \\ a_1' &= a_1 e^{-ikL} \\ a_2 &= i t_2 a_1' \\ a_3 &= r_2 a_1' \\ a_3' &= a_3 e^{-ikL} \\ a_4 &= i t_1 a_3' + r a_0 \end{aligned}\right\}$$

6 equations for 7 fields. Can solve if a_0 is known, or all fields in relation to a_0.

$$a_1' = a_1 e^{-ikL} = (i t_1 a_0 + r_1 a_3')e^{-ikL} = i t_1 e^{-ikL} a_0 + r_1 r_2 e^{-i2kL} a_1'$$

$$\Rightarrow a_1' = a_0 \frac{i t_1 e^{-ikL}}{1 - r_1 r_2 e^{-i2kL}}, \quad \frac{a_2}{a_0} = \frac{-t_1 t_2 e^{-ikL}}{1 - r_1 r_2 e^{-i2kL}}$$

Power:

$$P = P_0 \cdot \left| \frac{t_1 t_2 e^{-ikL}}{1 - r_1 r_2 e^{-ik2L}} \right|^2 = P_0 \frac{T_1 T_2}{1 + R_1 R_2 - r_1 r_2 \cos(2kL)}$$

Summary:

- learned coupling equations for simple optical components
- combined equations to compute fields in an optical cavity
- computed power transmitted by the cavity

Next:

Understand features of a cavity

Look at a Michelson interferometer